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Adaptive Singularity-Free Control Moment Gyroscopes

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Design considerations of agile, precise, and reliable attitude control for a class of small spacecraft and satellites using adaptive singularity-free control moment gyroscopes (ASCMG) are presented. An ASCMG differs from that of a conventional control moment gyroscopes (CMG) because it is intrinsically free from kinematic singularities and so does not require a separate singularity avoidance control scheme. Furthermore, ASCMGs are adaptive to the asymmetries in the structural members (gimbal and rotor) as well as misalignments between the center of mass of the gimbal and rotors. Moreover, ASCMG clusters are highly redundant to failure and can function as variable- and constant-speed CMGs without encountering singularities. A generalized multibody dynamics model of the spacecraft–ASCMG system is derived using the variational principles of mechanics, relaxing the standard set of simplifying assumptions made in prior literature on CMG. The dynamics model so obtained shows the complex nonlinear coupling between the internal degrees of freedom associated with an ASCMG and the spacecraft attitude motion. The general dynamics model is then extended to include the effects of multiple ASCMG, called the ASCMG cluster, and the sufficient conditions for nonsingular cluster configurations are obtained. The adverse effects of the simplifying assumptions that lead to the intricate design of the conventional CMG, and how they lead to singularities, become apparent in this development. A bare-minimum hardware prototype of an ASCMG using low-cost commercial off-the-shelf components is developed to show the design simplicity and scalability. A geometric variational integration scheme is obtained for this multibody spacecraft–ASCMG system for numerical and embedded implementation. Attitude pointing control of a CubeSat with three ASCMGs in the absence of external torques is numerically simulated to demonstrate the singularity-free characteristics and redundancy of the ASCMG cluster.

I. Introduction

A CONTROL moment gyroscope (CMG) is a momentum exchange device that can be used as an actuator for spacecraft attitude control [1], including attitude stabilization of agile spacecraft [2]. A typical CMG consists of a symmetric, balanced rotor flywheel that can spin about its axis, while this rotation axis is rotated on a plane perpendicular to a gimbal on which the rotor assembly is mounted; see Fig. 1. CMGs can be categorized as single/double gimbal control moment gyroscope (SGCMG/DGCMG). Further, a variable-speed control moment gyroscope (VSCMG) combines the features of a constant-speed SGCMG with a reaction wheel (RW), which has variable angular speed of the rotor. Defining features and comparisons between SGCMGs and DGCMGs are given in the book by Wie [3]. SGCMG can produce larger torques for the same actuator mass compared to reaction wheels [4,5]. However, because of the complex nonlinear dynamics, with inherent geometric singularities between the input (gimbal) space and output (momentum) space, SGCMGs are not as commonly applied as RWs. Singularity analysis and singularity avoidance steering laws have been studied for different CMG configurations [6]. The dynamics models of CMGs obtained in the prior literature are based on a set of simplifying assumptions that are listed in Sec. II and given in Refs. [3,7–9]. From here on, CMG is used synonymously with SGCMG in this paper. Variations in the inertia matrix of a spacecraft with CMGs under these assumptions were considered in [10].

The adaptive singularity-free control moment gyroscope (ASCMG) does away with the assumptions in the design and dynamics model used for standard CMGs. The resulting design is asymmetric, with the centers of mass (c.m.) of the gimbal structure and rotor (flywheel) structure not coinciding. It is adaptive because it can be operated in both variable-speed and constant-speed modes, and a cluster can provide singularity-free three-axis attitude control of the base spacecraft. To better understand the relations between control inputs and base-body rotational motion, the dynamics of a spacecraft with an ASCMG is developed first, using a variational approach in the framework of geometric mechanics. Thereafter, the ASCMG is specialized to the case that its rotor flywheel is operated at constant speed (i.e., in CMG mode). Because the configuration space of rotational motion of a spacecraft with internal actuators is a nonlinear manifold, the global dynamics of this system is treated using the formulation of geometric mechanics [11,12]. The treatment of the dynamics and control of a rigid body (spacecraft) with an internal actuated rotor (reaction wheel) has been provided in [12]. Prior work on control of gyroscopic systems applied to rigid-body attitude tracking and stabilization in the framework of geometric mechanics can be found in [13–15]. This work uses the global representation of the attitude of a spacecraft provided by the rotation matrix from the spacecraft base-body-fixed coordinate frame to the inertial coordinate frame. This approach is powerful and general enough to globally treat the dynamics of a spacecraft with one or more ASCMGs without simplifying assumptions.

II. Motivation

The ASCMG dynamics model presented here is based on the generalized VSCMG model detailed in [16], which is currently the only model that relaxes the commonly used assumptions that simplify the dynamics. For the sake of completeness, we reiterate these commonly used assumptions for VSCMGs and CMGs next.

Assumption 1: There is no offset between the rotor (flywheel) center of mass and the gimbal structure's center of mass.

Assumption 2: Both the gimbal and the rotor-fixed coordinate frames are their corresponding principal axes frames, and the rotor is axisymmetric. Both rotor and gimbal inertias are about their respective center of masses.

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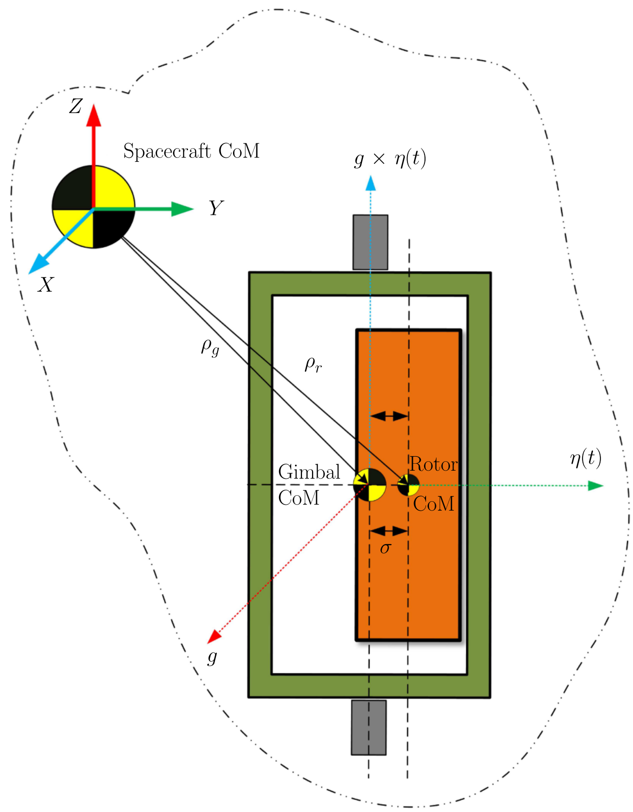


Fig. 1 Schematic of ASCMG showing an offset σ between the rotor and gimbal center of mass.

Assumption 3: The gimbal frame structure's angular momentum is "negligible" compared to that of the flywheel.

Assumption 4: The angular acceleration of the gimbal frame is negligible.

It is shown in [16] that these assumptions lead to the output torque of a CMG lying along the plane perpendicular to its gimbal axis. Because vectors lying on different planes in three-dimensional Euclidean space can be parallel, this leaves open the possibility that the output torque or angular momentum vectors generated by a set of three or more such CMGs do not span $\mathbb{R}^3$ at some instants. Indeed, this momentary (instantaneous) occurrence of a lack of spanning vectors leads to instantaneous singularities in the transformation from gimbal angular rates to torque generated on the spacecraft bus. At the instants when these singularities occur, an arbitrary torque vector on the spacecraft bus cannot be generated by the cluster of CMGs. Further, the occurrence of singularities cannot be removed by adding more CMGs or by arranging them in any other configuration. Besides this aspect of singularities in the transformation from gimbal rates to output torque (or output angular momentum), there are also difficulties associated with the manufacture of CMGs under the constraints posed by these assumptions. In particular, assumptions 1, 2, and 3 impose stringent conditions on the manufacturing, even if these assumptions are "approximately" satisfied. Assumptions 3 and 4 also impose constraints on the operations of CMGs. Although manufacturing tolerances may be relaxed for large CMGs operating in large spacecraft, these constraints become very stringent for small CMGs designed to operate in microspacecraft. The primary motivation for this work is to relax these restrictive assumptions to avoid the drawbacks associated with the use and manufacture of CMGs, especially for use in small satellites.

III. Spacecraft Dynamics with an Adaptive Singularity-Free Control Moment Gyroscope

The dynamics model of a spacecraft with a single ASCMG is formulated here. The gimbal axis of the ASCMG is perpendicular to the rotor's axis of rotation, which passes through its center of mass. There is a scalar offset along the rotor axis between the centers of mass of the gimbal and the rotor; this offset is denoted by σ.

A. Coordinate Frame Definitions

Consider a right-handed coordinate frame fixed to the center of mass of the spacecraft base body (spacecraft bus) that has an ASCMG as an internal attitude actuator. The attitude of the spacecraft is given by the rotation matrix from this base-body-fixed coordinate frame to an inertially fixed coordinate frame. Consider the gimbal of the ASCMG rotating about an axis fixed with respect to this spacecraft base-body-fixed coordinate frame. Let $g \in \mathbb{S}^2$ denote the fixed axis of rotation of the gimbal, and let $\alpha(t)$ represent the gimbal angle, which is the rotation angle of rotor axis η about the gimbal axis g. Here, $\mathbb{S}^n$ is an n-sphere defined by $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1}: \|x\| = r\}$, where r is the radius of the sphere. Then, $\eta(\alpha(t)) \in \mathbb{S}^2$ denotes the instantaneous direction of the axis of rotation of the rotor, which is orthogonal to g and depends on the gimbal angle $\alpha(t)$. Let $\theta(t)$ denote the instantaneous rotation angle of the rotor about its rotation axis $\eta(\alpha(t))$; this is the angle between the first axis of an ASCMG rotor-fixed coordinate frame and the gimbal axis, where the rotor axis forms the second coordinate axis of this coordinate frame. For an ASCMG, the angular speed of the rotor $\dot{\theta}(t)$ is time-varying by default, but it can also be set to constant speed corresponding to operation in CMG mode; details are provided in Sec. V. Both g and $\eta(\alpha(t))$ are expressed as orthogonal unit vectors in the base-body coordinate frame. Let σ be the distance of the center of rotation of the rotor from the center of mass of the gimbal; this distance is assumed to be along the vector $\eta(\alpha(t))$. Define an ASCMG gimbal-fixed coordinate frame with its first coordinate axis along g, its second axis along $\eta(\alpha(t))$, and its third axis along $g \times \eta(\alpha(t))$, to form a right-handed orthonormal triad of basis vectors as depicted in Fig. 1.

B. Rotational Kinematics

If $\eta(0)$ denotes the initial rotor axis direction vector with respect to the base-body frame, the rotor axis at time $t > 0$ is given by

$$\eta(\alpha(t)) = \exp(\alpha(t) g^{\times})\eta(0) \tag{1}$$

using Rodrigues's rotation formula. The time rate of change of the rotor axis $\eta(\alpha(t))$ is given by

$$\dot{\eta}(\alpha(t), \dot{\alpha}(t)) = \dot{\alpha}(t) g^{\times} \eta(\alpha(t)) \tag{2}$$

where the map $(\cdot)^{\times}: \mathbb{R}^3 \to \mathfrak{so}(3)$ is the vector space isomorphism $\in \mathbb{R}^3$ and $\in \mathfrak{so}(3)$, given by

$$g^{\times} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}^{\times} = \begin{bmatrix} 0 & -g_z & g_y \\ g_z & 0 & -g_x \\ -g_y & g_x & 0 \end{bmatrix}$$

Let R_g denote the rotation matrix from an ASCMG gimbal-fixed coordinate frame to the spacecraft base-body coordinate frame, which can be expressed as

$$R_g(\alpha(t)) = [\, g \quad \eta(\alpha(t)) \quad g \times \eta(\alpha(t)) \,] \tag{3}$$

The time derivative of R_g is then given by

$$\dot{R}_g(\alpha(t), \dot{\alpha}(t)) = \dot{\alpha}(t) R_g(\alpha(t)) e_1^{\times} \tag{4}$$

where $e_1 = [1 \quad 0 \quad 0]^T$, $e_2 = [0 \quad 1 \quad 0]^T$, and $e_3 = [0 \quad 0 \quad 1]^T$ are the standard basis vectors (expressed as column vectors) of $\mathbb{R}^3$. Because e_1 denotes the gimbal axis in the gimbal-fixed frame, the angular velocity of the gimbal-fixed frame with respect to the base body, expressed in the base-body frame, is

$$\omega_g(t, \dot{\alpha}) = \dot{\alpha}(t) g \tag{5}$$

Let $\theta(t)$ denote the instantaneous angle of rotation of the rotor about its symmetry axis $\eta(\alpha(t))$; this is the angle between the first axis of an ASCMG rotor-fixed coordinate frame and the gimbal axis, where the rotor axis forms the second coordinate axis of this coordinate frame. For operation in VSCMG mode, the angular speed of the rotor $\dot{\theta}(t)$ is time-varying, and a constant rotor rate corresponds to CMG mode operations; details are provided in Sec. V. The rotation matrix from this ASCMG rotor-fixed coordinate frame to the base-body coordinate frame is

$$R_r(\alpha(t),\theta(t)) = R_g(\alpha(t))\exp(\theta(t)e_2^\times) \tag{6}$$

The time rate of change of the rotation matrix R_r is obtained as follows:

$$\begin{aligned}
&\dot{R}_r(\alpha(t),\theta(t),\dot{\alpha}(t),\dot{\theta}(t))\\
&\quad = \dot{R}_g(\alpha(t),\dot{\alpha}(t))\exp(\theta(t)e_2^\times) + R_g(\alpha(t))\exp(\theta(t)e_2^\times)\dot{\theta}(t)e_2^\times,\\
&\quad = \dot{\alpha}(t)R_r(\alpha(t),\theta(t))(\exp(-\theta(t)e_2^\times)e_1)^\times + R_r(\alpha(t),\theta(t))\dot{\theta}(t)e_2^\times
\end{aligned}$$

using the fact that $R^T a^\times R = (R^T a)^\times$ for $R \in \mathrm{SO}(3)$ and $a \in \mathbb{R}^3$ [17]. It can be verified that the following holds for $\phi \in \mathbb{S}^1$:

$$\exp(\phi e_2^\times)e_1 = (\cos\phi)e_1 - (\sin\phi)e_3$$

Making use of this identity, the time derivative of R_r can be expressed as

$$\begin{aligned}
&\dot{R}_r(\alpha(t),\theta(t),\dot{\alpha}(t),\dot{\theta}(t))\\
&\quad = R_r(\alpha(t),\theta(t))\Big(\dot{\alpha}(t)((\cos\theta(t))e_1 + (\sin\theta(t))e_3) + \dot{\theta}(t)e_2\Big)^\times
\end{aligned}$$

Therefore, the angular velocity of the ASCMG rotor with respect to the base body, expressed in the base-body frame, is

$$\begin{aligned}
&\omega_r(\alpha(t),\dot{\alpha}(t),\dot{\theta}(t))\\
&\quad = R_r(\alpha(t),\theta)\Big(\dot{\alpha}(t)((\cos\theta(t))e_1 + (\sin\theta(t))e_3) + \dot{\theta}(t)e_2\Big)\\
&\quad = \dot{\alpha}(t)R_g(\alpha(t))e_1 + \dot{\theta}(t)R_g(\alpha(t))e_2 = \dot{\alpha}(t)g + \dot{\theta}(t)\eta(\alpha(t)) \quad (7)
\end{aligned}$$

Let $R(t)$ denote the rotation matrix from the base-body-fixed coordinate frame to an inertial coordinate frame. If $\Omega(t)$ is the total angular velocity of the base body expressed in the base-body frame, then the attitude kinematics of the spacecraft base body is given by

$$\dot{R}(t) = R(t)\Omega(t)^\times \tag{8}$$

Let ρ_g denote the position vector from the center of mass of the base body to the center of mass of the gimbal, expressed in the base-body frame. The position vector from the center of mass of the base-body to the center of mass of the rotor is therefore given by $\rho_r(\alpha(t)) = \rho_g + \sigma\eta(\alpha(t))$, in the base-body frame. These vectors expressed in the inertial frame are

$$r_g(R(t)) = R(t)\rho_g \quad \text{and} \quad r_r(R(t),\alpha(t)) = R(t)\rho_r(\alpha(t))$$

Thus, the inertial velocities of these centers of masses with respect to the base-body center of mass are given by

$$\begin{aligned}
\dot{r}_g(R(t),\Omega(t)) &= R(t)\Omega(t)^\times\rho_g \quad \text{and}\\
\dot{r}_r(R(t),\Omega(t),\alpha(t),\dot{\alpha}(t)) &= R(t)\Big(\Omega(t)^\times\rho_r(\alpha(t)) + \sigma\dot{\alpha}(t)g^\times\eta(\alpha(t))\Big)
\end{aligned} \tag{9}$$

The spacecraft with an ASCMG has five rotational degrees of freedom, which are described by the variables α, θ, and R. The configuration space Q of this system has the structure of a trivial principal (fiber) bundle with base space $B = \mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ and fiber $G = \mathrm{SO}(3)$ [18].

C. Formulation of the Equations of Motion

The equations of motion for a spacecraft with an ASCMG are obtained using variational mechanics, taking into account the geometry of the configuration space. Rigorous and comprehensive details of the derivation of kinematics and dynamics equation of a spacecraft with ASCMGs are given in Refs. [16,19,20]. For the sake of completeness, we provide some details of the obtained dynamics model.

The generalized model of an ASCMG consists of a rotor of mass m_r with inertia J_r resolved about its center of mass; the rotor is mounted on a gimbal structural frame of mass m_g with inertia J_g resolved about the point of intersection of the gimbal axis and the rotor axis. For notational convenience, the time dependence of variables is not explicitly denoted in the remainder of this paper, i.e., $\Omega = \Omega(t)$, $R_g(\alpha) = R_g(\alpha(t))$, $\dot{r}_r(\alpha,\dot{\alpha}) = \dot{r}_r(\alpha(t),\dot{\alpha}(t))$, etc.

The total rotational kinetic energy of a spacecraft with one ASCMG is

$$T(\gamma,\chi) = \frac{1}{2}\chi^T\mathcal{J}(\gamma)\chi \tag{10}$$

where,

$$\gamma = \begin{bmatrix}\alpha(t)\\ \theta(t)\end{bmatrix}, \quad \chi = \begin{bmatrix}\Omega(t)\\ \dot{\gamma}(t)\end{bmatrix} \quad \text{and} \quad \mathcal{J}(\gamma) = \begin{bmatrix}\Lambda(\gamma) & B(\gamma)\\ B(\gamma)^T & J_{\mathrm{gr}}(\gamma)\end{bmatrix} \tag{11}$$

Consider the spacecraft system to be in the absence of gravity. In this case, the Lagrangian of the spacecraft with an ASCMG is given by $\mathcal{L}(\gamma,\chi) = T(\gamma,\chi)$. Note that the Lagrangian is independent of the attitude R of the base body in this case. Define the following angular momentum quantities, which depend on the kinetic energy (10):

$$\Pi = \frac{\partial\mathcal{L}}{\partial\Omega} = \frac{\partial T}{\partial\Omega} = \overbrace{\Lambda(\gamma)\Omega}^{\text{Basebody momenta}} + \underbrace{B(\gamma)\dot{\gamma}}_{\text{ASCMG momenta}} \tag{12}$$

$$p = \frac{\partial\mathcal{L}}{\partial\dot{\gamma}} = \frac{\partial T}{\partial\dot{\gamma}} = B(\gamma)^T\Omega + J_{\mathrm{gr}}(\gamma)\dot{\gamma} \tag{13}$$

Associated inertia quantities occurring in Eqs. (11–13) are

$$\begin{aligned}
&\Lambda(\gamma) = \Lambda_0 + I_T(\gamma); \quad \Lambda_0 = J_b - (m_g + m_r)(\rho_g^\times)^2,\\
&I_T(\gamma) = J_c(\gamma) + I_c(\alpha) - m_r\sigma\eta(\alpha)^\times\rho_g^\times
\end{aligned} \tag{14}$$

$$J_c(\gamma) = R_g(\alpha)J_gR_g(\alpha)^T + R_r(\gamma)J_rR_r(\gamma)^T \tag{15}$$

$$I_c(\alpha) = -m_r\sigma\Big(\rho_g^\times\eta(\alpha)^\times + \sigma(\eta(\alpha)^\times)^2\Big) \tag{16}$$

$$B(\gamma) = \Big[\,(J_c(\gamma) + I_c(\alpha))g \quad R_rJ_rR_r^T\eta(\alpha)\,\Big] \in \mathbb{R}^{3\times 2} \tag{17}$$

$$I_r(\alpha) = -m_r(\sigma\eta(\alpha)^\times)^2, \qquad \text{and} \tag{18}$$

$$J_{\mathrm{gr}}(\gamma) = \begin{bmatrix} g^T(J_c(\gamma) + I_r(\alpha))g & g^TR_rJ_rR_r^T\eta(\alpha)\\ \eta(\alpha)^TR_rJ_rR_r^Tg & \eta(\alpha)^TR_rJ_rR_r^T\eta(\alpha)\end{bmatrix} \tag{19}$$

Here, J_b is the inertia of the spacecraft base body; J_g and J_r are the corresponding gimbal and rotor inertias, respectively. The constant part of total inertia in the base-body frame is denoted by Λ_0, whereas $I_T(\gamma)$ is the total time-varying part of inertia in the base-body frame. $J_c(\gamma)$ and $I_c(\alpha)$ are the offset-independent part of inertia and offset-dependent part of the ASCMG inertia in the spacecraft base-body frame, respectively. The equations of motion are obtained from Eqs. (12) and (13) by applying the Lagrange–d'Alembert principle:

$$\frac{\mathrm{d}\Pi}{\mathrm{d}t} = \Pi \times \Omega \tag{20}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\partial T}{\partial \gamma} + \tau \tag{21}$$

For an axisymmetric rotor rotating about its axis of symmetry, the Lagrangian is independent of the angle θ, and the scalar angular momentum p_θ would be conserved in the absence of any friction or damping torques acting on the rotor.

IV. Angular Momentum of the Generalized Adaptive Singularity-Free Control Moment Gyroscope Model

This section provides a generalized expression for angular momentum of the spacecraft due to an ASCMG, which is the second term in Eq. (12), without any of the assumptions used in the prior literature as listed in Sec. II. This expression is also derived in [16]. The momentum contribution from an ASCMG to the total angular momentum of the spacecraft is given by

$$u = B(\gamma)\dot{\gamma} = (J_c(\gamma) + I_c(\alpha))\dot{\alpha} g + R_r J_r R_r^T \dot{\theta} \eta(\alpha) \tag{22}$$

The inertia terms in Eqs. (15) and (16) are simplified to

$$J_c(\gamma) = -m_r \sigma \Big(\rho_g^\times \eta(\alpha)^\times + \sigma(\eta(\alpha)^\times)^2 \Big)$$

Therefore,

$$u = \dot{\alpha} J_c(\gamma) g + \dot{\alpha} I_c(\alpha) g + \dot{\theta} R_r J_r R_r^T \eta(\alpha) \tag{23}$$

Note that $R_r = R_g \exp(\theta e_2^\times)$. The first term on the expression for u can be simplified to

$$J_c(\gamma) g = R_g J_g e_1 + R_r J_r (\cos\theta e_1 + \sin\theta e_3) \tag{24}$$

The second term in Eq. (23) can be written as

$$\dot{\alpha} I_c(\alpha) g = m_r \sigma \Big[\rho_g^\times + \sigma \eta(\alpha)^\times \Big] \dot{\eta}(\alpha) \tag{25}$$

and the last term of Eq. (23) is

$$R_r J_r R_r^T \eta(\alpha) = R_r J_r e_2 \tag{26}$$

Using Eqs. (24–26), expression (23) for u leads to

$$\begin{aligned} u = \dot{\alpha} \Big(& [R_g J_g e_1 + R_r J_r (\cos\theta e_1 + \sin\theta e_3)] \\ & + m_r \sigma [\rho_g^\times + \sigma \eta(\alpha)^\times] g^\times \eta(\alpha) \Big) + \dot{\theta} R_r J_r e_2 \end{aligned} \tag{27}$$

This angular momentum expression [Eq. (27)] for the generalized ASCMG model can be reduced to an ASCMG with constant-speed rotor, as described next in Sec. V.

V. Reduction to Constant-Speed Adaptive Singularity-Free Control Moment Gyroscope

An ASCMG can be operated as a CMG by spinning the rotor flywheel at a constant angular velocity ($\dot{\theta} = c; \ddot{\theta} = 0$). Equation (27) can be reexpressed as

$$\begin{aligned} u &= [\, D(\alpha,\theta) \quad E(\alpha) \,] \begin{bmatrix} \dot{\alpha} \\ c \end{bmatrix}, \quad \text{or} \\ u - cE(\alpha) &= \dot{\alpha} D(\alpha,\theta), \\ \text{where,} \quad D(\alpha,\theta) &= [R_g J_g e_1 + R_r J_r (\cos\theta e_1 + \sin\theta e_3)] \\ &\quad + m_r \sigma [\rho_g^\times + \sigma \eta(\alpha)^\times] g^\times \eta(\alpha), \\ \text{and} \quad E(\alpha) &= R_r J_r e_2 \end{aligned} \tag{28}$$

For the case of multiple ($n \geq 3$) CMGs, we have an ASCMG cluster whose momentum contribution from Eqs. (22) and (27) is

$$\begin{aligned} u &= \sum_{i=1}^{n} D^i(\alpha^i, \theta^i) \dot{\alpha}^i + \sum_{i=1}^{n} E^i(\alpha^i) c^i \\ u &= \mathcal{D}(\vartheta, \Theta) \dot{\vartheta} + \mathcal{E}(\vartheta) C, \\ \bar{u} &:= u - \mathcal{E}(\vartheta) C = \mathcal{D}(\vartheta, \Theta) \dot{\vartheta} \end{aligned} \tag{29}$$

Here, u is the total angular momentum from the ASCMG cluster, and

$$\Theta = [\, \theta^1 \quad \theta^2 \quad \cdots \quad \theta^n \,]^T \in \mathbb{R}^n \tag{30}$$

$$\vartheta = [\, \alpha^1 \quad \alpha^2 \quad \cdots \quad \alpha^n \,]^T \in \mathbb{R}^n \tag{31}$$

$$\mathcal{E} = \Big[E^1(\alpha^1) \| E^2(\alpha^2) \| \ \cdots \ \| E^n(\alpha^n) \Big] \in \mathbb{R}^{3\times n} \tag{32}$$

$$\mathcal{D} = \Big[D^1(\alpha^1, \theta^1) \| D^2(\alpha^2, \theta^2) \| \ \cdots \ \| D^n(\alpha^n, \theta^n) \Big] \in \mathbb{R}^{3\times n} \tag{33}$$

$$\text{and,} \quad C = \dot{\Theta} = [\, c^1 \quad c^2 \quad \cdots \quad c^n \,]^T \in \mathbb{R}^n \tag{34}$$

Note that, in the traditional dynamics model for CMGs, the matrix $\mathcal{D}$ in Eq. (33) does not depend on θ^i because the rotor is assumed to be axisymmetric. Moreover, the angular momentum generated by the CMGs does not depend on their location inside the spacecraft body (denoted by ρ_g^i) if there is no offset between the centers of mass of the gimbal frame and the rotor/flywheel.

VI. Attitude Control Using Adaptive Singularity-Free Control Moment Gyroscope

For spacecraft attitude stabilization or tracking using ASCMG clusters operated in CMG mode, one needs to ensure that the angular velocity Ω, or alternately the spacecraft base-body momentum Π_b, is controlled by the internal momentum u, which depends on the gimbal rates $\dot{\vartheta}$. One can express the total angular momentum of a spacecraft with n ASCMGs using Eq. (12) as follows:

$$\begin{aligned} \Pi = \Pi_b + u, \quad & \text{where } \Pi_b = \Lambda(\vartheta, \Theta)\Omega, \\ \text{and} \quad u = \mathcal{D}(\vartheta, \Theta)\dot{\vartheta} + & \mathcal{E}(\vartheta) C \end{aligned} \tag{35}$$

Therefore, the attitude dynamics of the spacecraft with a cluster of ASCMGs is

$$\dot{\Pi}_b = \Pi_b \times \Omega + (u \times \Omega - \dot{u}) = \Pi_b \times \Omega + \tau_{\mathrm{cp}} \tag{36}$$

where $\tau_{\mathrm{cp}} = u \times \Omega - \dot{u}$ is the control torque generated by the "internal" momentum u from the ASCMGs. Note that, from Eq. (36) and the attitude kinematics [Eq. (8)], the total momentum in an inertial frame $\Pi_I = R\Pi = R(\Pi_b + u)$ is conserved. There are several control problems of interest for a spacecraft with n ASCMGs. For most space telescopes and Earth observation applications in low Earth orbit, spacecraft attitude maneuvers require high agility and pointing precision. Here, a pointing maneuver of an agile spacecraft with ASCMG cluster is considered.

The objective of a pointing maneuver for spacecraft is to point a body-fixed imaging, communication, or other instrument in a specified direction in a reference frame. To point a spacecraft to a desired attitude, a continuous feedback controller given by $\tau_{\text{cp}}(R, \Omega)$: $\mathrm{SO}(3) \times \mathbb{R}^3 \to \mathbb{R}^3$ is selected that asymptotically stabilizes the desired attitude $R_d \in \mathrm{SO}(3)$ with zero angular velocity. The continuous feedback control torque for this system is given by [21]

$$\tau_{\text{cp}} = -L_v \Omega - L_p \Xi(R) \tag{37}$$

where $L_v, L_p \in \mathbb{R}^{3\times 3}$ are positive–definite matrices and

$$\Xi(R) \triangleq \sum_{i=1}^{3} a_i e_i \times (R_d^T R.e_i) \tag{38}$$

with $[\, e_1 \quad e_2 \quad e_3 \,] \equiv I_{3\times 3}$ and $a = [\, a_1 \quad a_2 \quad a_3 \,]^T$, where a_1, a_2, and a_3 are distinct positive integers. This control law asymptotically stabilizes the attitude and angular velocity states to $(R, \Omega) = (R_d, 0)$, for a given desired attitude, $R_d \in \mathrm{SO}(3)$ [21].

The control inputs for a spacecraft with an ASCMG cluster are the CMG gimbal rates, obtained by taking the pseudoinverse of $\mathcal{D}$ in Eq. (29). This gives the gimbal rates $\dot{\alpha}^i$ for all CMGs with constant rotor rates $\dot{\theta}^i = c^i$ as

$$\dot{\vartheta} = \mathcal{D}^{\dagger} \bar{u}, \qquad \text{where} \quad \mathcal{D}^{\dagger} = \mathcal{D}^T (\mathcal{D}\mathcal{D}^T)^{-1} \tag{39}$$

and $\bar{u} = u - \mathcal{E}(\vartheta) C$, where u is obtained by integrating $\tau_{\text{cp}} = u \times \Omega - \dot{u}$. The CMGs need to be oriented in a way such that $\mathcal{D} \in \mathbb{R}^{3\times n}$ in Eq. (33) always has full row rank for $n \geq 3$ (i.e., $\mathcal{D}\mathcal{D}^T$ is nonsingular). Necessary and sufficient conditions for $\mathcal{D}$ to be nonsingular are given in Sec. VII.

Additionally, the CMGs can be maintained at their nominal rates $\dot{\vartheta}^{\star}$, without exerting any torque on the spacecraft bus, by operating the CMGs in the null space of $\mathcal{D} \in \mathbb{R}^{3\times n}$, called null motions of the ASCMG cluster. When $n \geq 3$ for an ASCMG cluster, Eq. (39) can be modified to include null motions as follows:

$$\dot{\vartheta} = \mathcal{D}^{\dagger} \bar{u} + (I_{n\times n} - \mathcal{D}^{\dagger}\mathcal{D}) \dot{\vartheta}^{\star} \quad \text{for } \vartheta^{\star} \in \mathbb{R}^n \tag{40}$$

VII. Singularity Analysis of Adaptive Singularity-Free Control Moment Gyroscope Cluster

Expressions for the angular momentum contribution from an ASCMG cluster and the time derivative of this angular momentum are provided here. As can be seen from these expressions, the "control influence" matrices are nonsingular for $n \geq 3$ when either this angular momentum or its time derivative are considered as control inputs, under simple and easy-to-satisfy conditions on the gimbal axis orientations. This shows that, with as few as three of these CMGs, one can obtain a desired output angular momentum or output torque from the CMGs, provided that angular rate and angular acceleration constraints on the gimbal and rotor are satisfied.

Consider n CMGs, satisfying assumption 2 in Sec. II, in which case the angular momentum contribution of each CMG can be simplified from that given by Eq. (28). Let J_{g1}^k denote the inertia of the gimbal frame of the kth CMG about the gimbal axis (its symmetry axis), and let $J_r^k = \mathrm{diag}(J_{r1}^k, J_{r2}^k, J_{r1}^k)$ denote the inertia matrix of the axisymmetric rotor of the kth CMG, where $k = 1, \ldots, n$ for $n \geq 3$. Then, Eq. (28) for the momentum output of the CMGs in the base-body coordinate frame specializes to

$$\begin{aligned} &\mathcal{D}(\vartheta)\dot{\vartheta} + \mathcal{E}(\vartheta) C = u, \\ &\text{where } \mathcal{D}(\vartheta) = \left[D^1(\alpha^1) \| D^2(\alpha^2) \| \ \cdots \ \| D^n(\alpha^n) \right] \in \mathbb{R}^{3\times n} \\ &\mathcal{E}(\vartheta) = \left[J_{r2}^1 \eta^1(\alpha^1) \| J_{r2}^2 \eta^2(\alpha^2) \| \ \cdots \ \| J_{r2}^n \eta^n(\alpha^n) \right] \in \mathbb{R}^{3\times n} \\ &D^k(\alpha^k) = (J_1^k + m_r^k \sigma_k^2) g^k + m_r^k \sigma_k (\rho_g^k)^{\times} (g^k)^{\times} \eta^k(\alpha^k) \\ &\text{and} \quad J_1^k = J_{g1}^k + J_{r1}^k \end{aligned} \tag{41}$$

The following result easily follows from the structure of the matrix $\mathcal{D}$ in the preceding expression.

Theorem 1: For a spacecraft with three or more CMGs ($n \geq 3$) satisfying assumption 2 as stated earlier, the control influence matrix $\mathcal{D}(\vartheta)$ defined by Eq. (41) has full rank if

$$J_1^k + m_r^k \sigma_k^2 + m_r^k \sigma_k (\rho_g^k)^T \eta^k(\alpha^k) \neq 0, \quad (\rho_g^k)^T g^k = 0 \tag{42}$$

and g^k span $\mathbb{R}^3$ for $k \in \{1, \ldots, n\}$.

Proof: The expression for $D^k(\alpha^k)$ in Eq. (41) can be simplified to

$$\begin{aligned} D^k(\alpha^k) = &\left(J_1^k + m_r^k \sigma_k^2 + m_r^k \sigma_k (\rho_g^k)^T \eta^k(\alpha^k) \right) g_k \\ &- m_r^k \sigma_k \left((\rho_g^k)^T g^k \right) \eta^k(\alpha^k) \end{aligned} \tag{43}$$

using the vector triple product identity. Therefore, if the relations in Eq. (42) hold, then $D^k(\alpha^k)$ is parallel to g^k. Without loss of generality, let g^1, g^2, and g^3 span $\mathbb{R}^3$. Then, the vectors $D^k(\alpha^k)$ for $k = 1, 2, 3$ span $\mathbb{R}^3$, and $\mathcal{D}(\vartheta)$ has full rank. □

The momenta of the general ASCMG model in Eq. (27) can be further reduced to that of the traditional CMG model by imposing assumptions 1–4, in sequence. The resulting simplified momenta expression for that of a traditional CMG is obtained as

$$u = \dot{\theta} J_{r2} \eta(\alpha) \tag{44}$$

The time derivative of $u(t)$, which is part of the control torque on the spacecraft due to the simplified VSCMG, is evaluated under these assumptions. This is evaluated to be

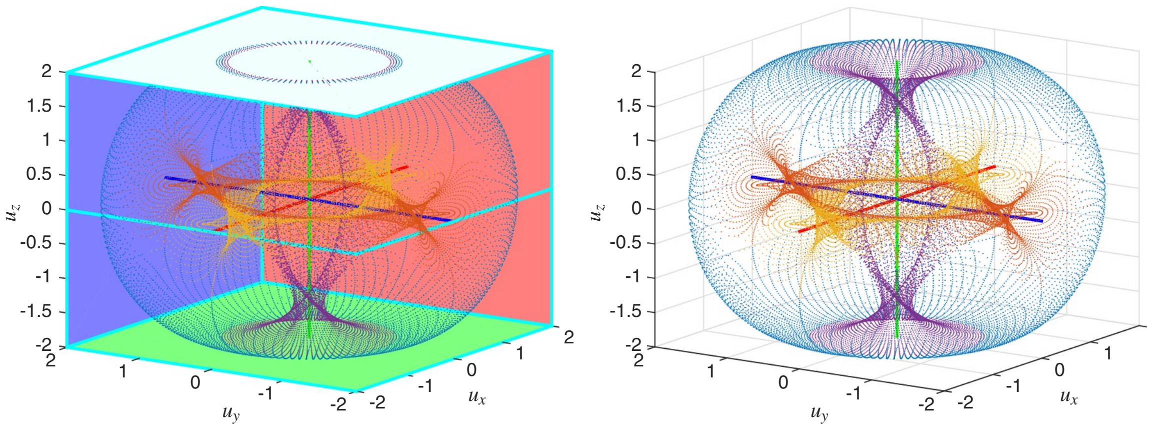
$$\tau_u = \frac{\mathrm{d}u}{\mathrm{d}t} = \ddot{\theta} J_{r2} \eta(\alpha) + \dot{\alpha} \dot{\theta} J_{r2} g^{\times} \eta(\alpha) \tag{45}$$

which shows that, under these additional assumptions, this torque component is on the plane normal to the gimbal axis g. Further, the speed of the rotor $\dot{\theta}$ is kept constant as in a standard CMG. Then, the time rate of change of u_{CMG} is given by

$$\tau_u = \frac{\mathrm{d}u}{\mathrm{d}t} = \dot{\theta} \dot{\alpha} J_{r2} g^{\times} \eta(\alpha) = \kappa \dot{\alpha} g^{\times} \eta(\alpha) \tag{46}$$

where $\kappa = \dot{\theta} J_{r2}$ is a constant scalar. This corresponds to the well-known model for standard CMGs where the control torque generated is known to be normal to the gimbal and rotor axes.

Several observations can be made as a result of the analysis leading to this main result. It is interesting to compare the preceding result with the corresponding result for the generalized VSCMG model given in [16]. In both cases, the generalized model that relaxes the restrictive assumptions makes it easy to avoid singularities in the transformation from gimbal rates (gimbal and rotor rates in the case of VSCMGs) to the angular momentum or torque produced on the spacecraft bus. However, in the case of VSCMGs, only two VSCMGs with the generalized model are required, whereas at least three CMGs are required to achieve singularity-free control for the spacecraft. It is also noteworthy that the preceding statement gives only a sufficient condition for singularity-free three-axis attitude control using CMGs. One may locate and orient the CMGs without satisfying Eq. (42) and still be able to avoid singularities and provide three-axis attitude control. In fact, it is clear from the preceding proof that this can be done even if only two of the CMGs satisfy the second



a) ASCMG momentum envelope, (27) **b) Conventional CMG momentum envelope, (44)**

Fig. 2 Total angular momentum envelope of ASCMG and conventional CMG.

equality in Eq. (42). Theorem 1 holds even with assumption 1 because it is clear that conditions (42) are trivially satisfied if $\sigma_k = 0$. However, having nonzero offsets ($\sigma_k \neq 0$) between gimbal and rotor centers of mass can provide additional control authority to the CMGs. All of the assumptions listed in Sec. II need to hold for singularities to appear in the relation between gimbal rates and output momentum (or torque) for CMGs. In particular, the CMG momentum expression given in Eq. (44) as the result of assumptions 3 and 4 is necessary for such singularities to appear. The momentum envelope of four such standard CMGs in pyramid configuration with the ubiquitous skew angle of 54.73 deg leads to the singular surface as shown in Fig. 2b because the angular momentum component of each CMG is acting only along the rotor axis $\eta(\alpha)$ and thus cannot span $\mathbb{R}^3$ along the gimbal axis. On the other hand, the momentum envelope generated by three ASCMGs in an orthogonal configuration spans entire $\mathbb{R}^3$ as shown in Fig. 2a due to the direct contribution from gimbal momentum (cube) enclosing the rotor momentum. These assumptions along with assumption 1 are particularly restrictive when it comes to manufacturing CMGs, thereby increasing the cost for manufacturing. Therefore, the design and generalized model of the ASCMG cluster given here is useful not only to avoid singularities (and therefore high angular rates) in the control logic but also to decrease manufacturing costs.

VIII. Design Considerations of Adaptive Singularity-Free Control Moment Gyroscope

In spite of a CMG's torque amplification characteristics, the practical application of CMGs in spacecraft attitude control is limited only to medium and large spacecraft. Besides, the inherent problem of kinematic singularities faced by the standard CMG design is due to the simplified dynamics model, which also lead to strict constraints on the mechanical design and manufacturing. The standard CMG design requires ultra precision machining and balancing because of the simplifying assumptions imposed in their dynamics model, as stated in Sec. II. It is very challenging in manufacturing technology to completely align the principle axis of inertia and the axis of rotation of a rotor. Even if the CMGs are manufactured with a high degree of precision, it may lose its alignment after repeated usage and eventually lead to malfunctions or even inoperability. The strict requirement of symmetry in the CMG's movable structures (i.e., axisymmetric gimbal and rotor) leads to manufacturing complexities and increase in the total mass budget. Also, in the traditional CMG operations, the rotor flywheels are allowed to spin at a very high angular rate, whereas the gimbal is rotated at a very slow rate, such that the gimbal rate is almost negligible compared to that of the rotor rate ($\dot{\alpha} \lll \dot{\theta}$). Mechanically, to achieve this condition, the CMG gimbal frame and bearing should be rigid enough to take all the resultant force/torque generated by the high-speed flywheel. When the principal axes of inertia of the CMG's high-speed flywheel

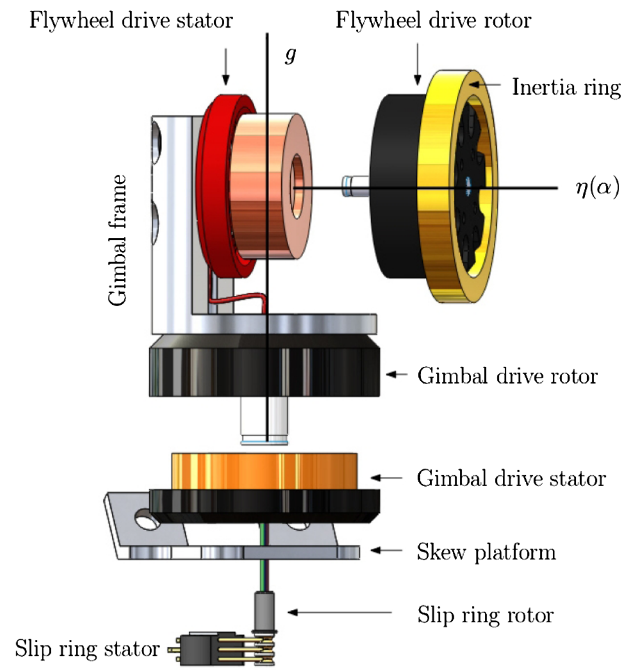


Fig. 3 Exploded view of ASCMG showing functional components.

deviate from the axis of rotation, there can be radial forces that eventually build up, leading to possible failure of the rotor drive.

A simple ASCMG hardware design is shown in Fig. 3, which is made possible by the generalized VSCMG dynamics model presented in [16] that is applicable to this design. Note that this design violates some of the simplifying assumptions listed in Sec. II. The electromechanical design presented here can be operated in both VSCMG and CMG modes. In traditional CMG design, the dynamics model is often simplified, and it is compensated with complex design/manufacturing constraints, whereas here with the relaxing of assumptions in the ASCMG dynamics formulation, the mechanical design is relatively much simpler, at the cost of a more complex nonlinear dynamics model. The design philosophy of the ASCMG is presented here in the following discussion.

A. Adaptability

ASCMGs are adaptive to misalignments within the hardware design of the ASCMG and between the spacecraft-ASCMG system. With two adaptive parameters incorporated in the general dynamics model of ASCMG, one can manage the misalignments, without requiring any mechanical modification to the spacecraft–ASCMG system. These adaptive parameters are discussed as follows.

1. Rotor Offset σ

The ASCMG hardware design is alignment-free, i.e., a scalar offset σ between the rotor flywheel's c.m. and the gimbal axis g, along the rotor axis η is considered ($\sigma \neq 0$). Aligning the flywheel–gimbal system in traditional CMG design is considered to be a crucial requirement for fault-free operations. This precise alignment may falter due to manufacturing deficiencies, after extended hours of continuous usage, and/or due to structural vibrations. Under such a situation, a microspacecraft with traditional CMGs can lose pointing accuracy [16], and the attitude control system will deteriorate in performance. Although the perfect alignment assumption simplifies the dynamics model, it considerably increases the manufacturing complexities besides leading to kinematic singularities that have to be accounted for in the control software. In case of an ASCMG-based attitude control system, this misalignment is deliberately introduced and accounted for by the control software. In addition to the adaptability and lack of kinematic singularities, the rotor offset in the ASCMG design can provide higher control authority [16,19,20,22].

2. Location of Adaptive Singularity-Free Control Moment Gyroscope in Spacecraft Bus, ρ_g

The center of mass of a spacecraft with an ASCMG cluster can be calibrated with limited accuracy during ground testing and can change further in orbit due to expulsion of fuel mass or reconfiguration. This in turn can reduce the attitude and orbit control precision. Online c.m. estimation schemes can be employed to estimate and update the location of the center of mass [23]. Revising the attitude control law based on the changing c.m. is important for precise control. The ASCMG design and associated control laws can adapt to variations in the spacecraft c.m. location, which is accounted for by the ρ_g parameter. This parameter denotes the position vector from the c.m. of the spacecraft base body (bus) to the gimbal c.m.

B. Scalability

The traditional CMG design is not scalable because misalignments between the c.m. of the CMG rotor and the gimbal axis cannot be tolerated for CMGs to be used for small spacecraft (micro- and nanosatellites). The ASCMG design and dynamics are highly scalable because this design considers parameters that are neglected in the traditional CMG design.

C. Asymmetric Structure

It can be noted that the VSCMG gimbal frame is not symmetric about the gimbal axis g, such that the gimbal inertia J_g includes off-diagonal components as well. To achieve precise axisymmetry of the flywheel about its rotational axis $\eta(\alpha)$, the flywheel should be dynamically balanced. The VSCMG dynamics model presented here does not demand such precise symmetry. Also, gimbal and rotor inertia need not be resolved about their respective center of mass (i.e., $J_g \neq J_{g\mathrm{CoM}}$ and $J_r \neq J_{r\mathrm{CoM}}$). Both the gimbal- and the rotor-fixed coordinate frames need not be their corresponding principal axes frames, and the rotor need not be perfectly axisymmetric. Additionally, relaxing these stringent requirements reduces the demands on sophisticated machining and dynamic balancing, which in turn is associated with decreased manufacturing costs.

D. Nominal Adaptive Singularity-Free Control Moment Gyroscope Rates

Sensorless brushless dc motors of outrunner type are used here as ASCMG direct drives (gimbal and flywheel drive). The direct drive arrangement eliminates the effect of multiple-element torsional oscillation system and minimizes the mass without compromising the performance. The gimbal and flywheel angular rates are maintained at their nominal rates, which enables the use of the back Electromotive force sensing method for sensorless commutation and does not require high-resolution rotational encoders as in the case of traditional CMG gimbal drives. Mechatronics architecture of the ASCMG cluster is given in [24].

E. Reliability

A minimalistic cluster of three ASCMGs in tetrahedron configuration can be used for reliable three-axis attitude control because it can be operated in both variable- and constant-speed rotor modes. In the event of an ASCMG hardware failure (gimbal/flywheel motors or electronics), the ASCMG cluster can still perform three-axis attitude control when operated in VSCMG mode with two ASCMGs. A simple hybrid control law can switch the ASCMG between VSCMG, CMG, and RW modes.

A miniature ASCMG hardware based on these design considerations is prototyped as in Fig. 4a, which can be used for experimental validation of the aforementioned cubesat attitude control. A highly redundant three-ASCMG tetrahedron cluster that can easily fit in $(1/2)U$ cubesat volume can be used to control a 1U to 6U cubesat, as shown in Fig. 4b, and has been prototyped using commercial off-the-shelf components.

IX. Results and Discussion

An agile and precise spacecraft attitude pointing maneuver is considered, with three ASCMGs ($n = 3$), each of mass 87 g, with corresponding gimbal axes (g_1, g_2, g_3) normal to the three faces of a tetrahedron at an angle of 54 deg and the fourth face of the tetrahedron mounted onto the spacecraft base body of mass 6 kg. The geometric variational integrator (GVI) obtained in [16] for a spacecraft with generalized VSCMGs gives the discrete-time evolution of the spacecraft attitude and angular momentum:

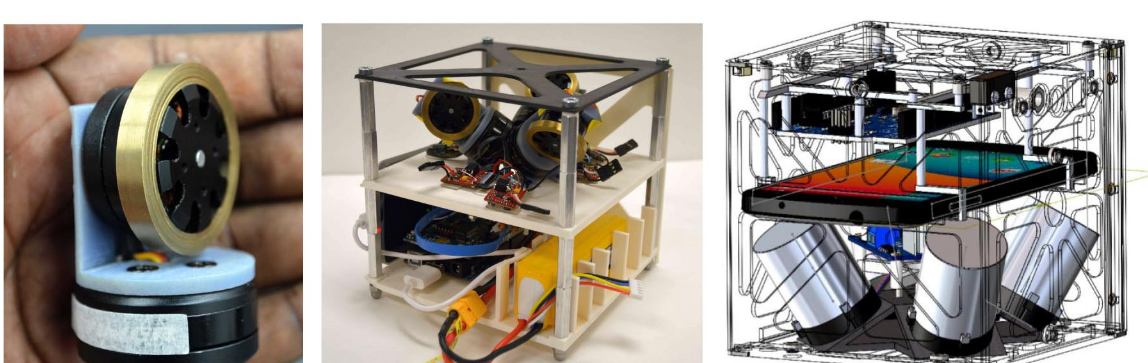
$$R_{i+1} = R_i F_i, \quad \Pi_{b,i+1} = F_i^T \Pi_{b,i} + F_i^T u_i - u_{i+1} \tag{47}$$

The GVI algorithm is modified appropriately for the case of a spacecraft with three or more CMGs as follows [25].

1) Initialize $F_i = \exp(h\Omega_i^\times)$, $\vartheta_{i+1} = h\dot{\vartheta}_i + \vartheta_i$, and $\Theta_{i+1} = hC + \Theta_i$.

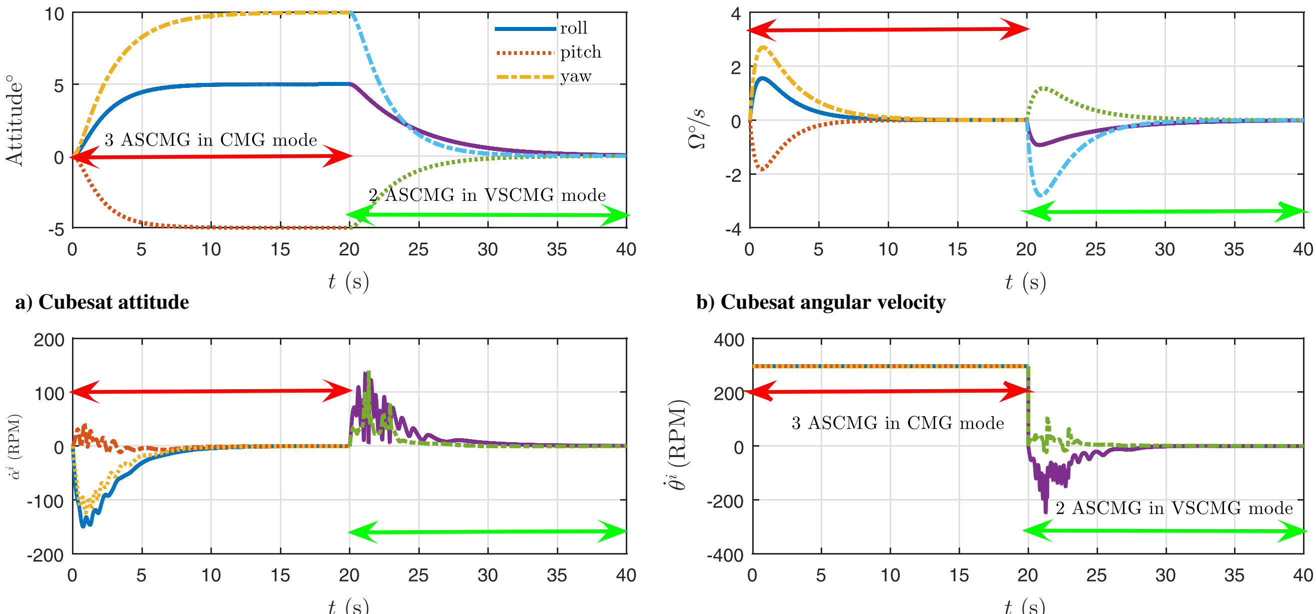
2) Compute $\mathcal{D}_{i+1}$, $\mathcal{E}_{i+1}$, and Λ_{i+1} as functions of ϑ_{i+1} and Θ_{i+1}.

3) Integrate $u_{i+1} = F_i^T u_i - h\tau_{\mathrm{cp},i}$.



a) b) c)

Fig. 4 Representations of a) unit ASCMG prototype, b) prototype of three-ASCMG cluster in 1U cubesat, and c) four-ASCMG pyramid cluster.



a) Cubesat attitude **b) Cubesat angular velocity**

c) Gimbal rates, rpm **d) Rotor rates, rpm**

Fig. 5 Numerical simulation results of an agile 1U cubesat with three ASCMGs.

4) Compute $\bar{u}_{i+1} = u_{i+1} - \mathcal{E}_{i+1}C$.
5) Update CMG gimbal rates one time step forward as

$$\dot{\vartheta}_{i+1} = \mathcal{D}_{i+1}^{\dagger}\bar{u}_{i+1} + (I_{n\times n} - \mathcal{D}_{i+1}^{\dagger}\mathcal{D}_{i+1})\dot{\vartheta}^{\star}$$

6) Obtain $\Pi_{b,i+1}$ from Eq. (47), and $\Omega_{i+1} = \Lambda_{i+1}^{-1}\Pi_{b,i+1}$.
7) Loop through steps 1 to 6 for all i.

This algorithm provides an explicit geometric variational integrator, which is used to numerically simulate the control problem of interest, while preserving the geometry of the configuration space.

To demonstrate the agility, precision, and reliability of the developed ASCMG cluster, a control objective of pointing the spacecraft to the desired attitude for a rest-to-rest maneuver is numerically simulated. A 1U CubeSat with three ASCMGs providing the attitude actuation in CMG mode for the first 20 s, then switching to the VSCMG mode at the instant of an assumed hardware failure in one of the ASCMG units, is considered. The simulation result shows that the fully functional ASCMG cluster, operated in CMG mode for the first 20 s, reoriented the spacecraft from rest to the desired attitude without singularities. With an assumed hardware failure in one unit, the cluster of three and the control scheme switch to operate the ASCMGs in VSCMG mode, which requires only two functional ASCMGs to bring the spacecraft back to its initial rest attitude, as shown in Figs. 5a and 5b. The gimbal and rotor rates for the three ASCMGs are shown in Figs. 5c and 5d, respectively. It is to be noted that the practical implementation of the control schemes and ASCMG design proposed here require the application of appropriate lower-level control inputs to the motors to implement the required gimbal and flywheel rates. Any loss in accuracy in implementing these required rates at the lower level will lead to corresponding loss in performance in the higher-level attitude control scheme.

X. Conclusions

In this paper, the adaptive singularity-free control moment gyroscope (ASCMG) is introduced, and the dynamics model of a spacecraft with an ASCMG is derived. This dynamics model is then generalized to a spacecraft with n ASCMGs. The ASCMG can operate in both VSCMG and CMG modes. Sufficient conditions for singularity-free operation of a cluster of ASCMGs in CMG mode is presented. A functional hardware design of a three-ASCMG cluster based on the developed design and dynamics model is prototyped. Agility, precision, and reliability of the ASCMG cluster is numerically validated for a cubesat. The adaptive and singularity-free characteristics of the ASCMG cluster are demonstrated through simulation. Experimental validation of the ASCMG prototype developed here will be carried out in the future.

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